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't Hooft Vortices on D-branes

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Abstract: The point where a D2-brane intersecting a stack of D2-branes is proposed as a candidate for the 't Hooft vortex in the world-volume theory of N D2-branes. This straightforwardly generalizes to D3-branes, where a vortex line is generated by the intersection. Similarly, there are such objects on M-branes. We use Maldacena's conjecture to compute the static potential between a vortex and an anti-vortex in each case, in the large N limit.

Keywords: D-branes, M-Theory, Brane Dynamics in Gauge Theories.

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1. Introduction

't Hooft in [1] introduced in a nonabelian gauge theory a new variable dual to the Wilson loop. If the Wilson loop is regarded as the usual order parameter, the dual variable may be regarded as a disorder parameter. Thus the dual variable is also useful in characterizing the phases of the nonabelian gauge theory.

There are two possible interpretations of the usual Wilson loop, depending on whether it is temporal or it is spatial. In the temporal case, the loop may be explained as traced out by a charge. For instance, if the charge is in the fundamental representation R of the gauge group $SU(N)$, then the corresponding Wilson loop is just

$$W(C)=\mathrm{tr}_R P e^{i\int_C A}\,, \tag{1.1}$$

where the trajectory C is parametrized by time. If the loop is spatial, the operator may be taken as a creation operator for a string-like object. Indeed there is an electric flux along the loop in this case.

The disorder parameter is a scalar field in $2+1$ dimensions, and a loop variable in $3+1$ dimensions. In the first case, there is an exchange relation

$$W(C)\phi(x)=e^{\frac{2\pi i}{N}}\phi(x)W(C)\,, \tag{1.2}$$

where again we assumed gauge group $SU(N)$. Note that loop C surrounds point x exactly once. If C and x are not "linked", there is no extra phase in the exchange relation. The effect of operator $\phi(x)$ is to introduce a gauge transformation which is singular at point x. Namely, if one follows a loop surrounding x, the gauge transformation is multi-valued: $G\to e^{2\pi i/N}G$. The phase factor belongs to the center Z_N of group $SU(N)$, therefore introduces no effects on fields in the adjoint representation.

The analogue of $\phi(x)$ in $3+1$ dimensions is a loop variable. For a loop C, denote this operator by $T(C)$. If C and C' have link number 1, then (1.2) generalizes to

$$T(C)W(C') = e^{\frac{2\pi i}{N}} W(C')T(C) \,. \tag{1.3}$$

We call $T(C)$ the 't Hooft loop operator.

One might speculate that the 't Hooft loop in a $3+1$ dimensional gauge theory ought to correspond to a loop traced out by a monopole. There are doubts whether this can be true. Indeed, a monopole is nonlocal with respect to an electric charge. It is so only when the monopole and the electric charge live in the same $U(1)$ group. Now for $SU(N)$, there are $N-1$ "fundamental" monopoles. If we were to construct a loop variable associated to monopoles, we must include all of them, just as in the case we construct the Wilson loop. Since different electric charges and different monopoles are not all nonlocal with respect to one another, it is hard to imagine the exchange relation (1.3) will result from this construction. Another problem associated with this is, if a trace of monopole is the 't Hooft loop, what object generates $\phi(x)$ in a $2+1$ gauge theory? Similar to the case of the spatial Wilson loop, the spatial 't Hooft loop on D3-branes then would have to be interpreted as an operator creating a trapped D-string. By T-duality, a vortex on D2-branes would have to be interpreted as a trapped D0-brane. It is well-known that a D0-brane, once trapped in D2-branes, can not be localized. We conclude that a D0-brane can not be a candidate for a vortex, hence the loop traced by a monopole can not be a candidate for a 't Hooft loop.

Our construction depends crucially on the existence of the adjoint scalars in the super Yang-Mills theory. It is possible to construct the 't Hooft loop operator in a gauge theory without adjoint Higgs fields, as done in [2].

In the next section, we propose that there is a natural realization of 't Hooft vortex in the world-volume theory of D2-branes, it is just given by the touch point on which an orthogonal D2-brane intersects the stack of D2-branes in question. Our construction is a simple generalization of an observation made in [3]. We argue that precisely for an operator which acts as the creation operator for this touch point, the exchange relation (1.2) holds. The generalization of this to a stack of D3-branes is obvious: The 't Hooft loop is just the loop along which an orthogonal D3-brane intersects the stack of D3-branes. In sect. 3, we use Maldacena's conjecture to compute the static potential between a vortex and an anti-vortex, in the large N limit. This calculation is generalized to D3-branes in sect. 4, and to M-branes in sect. 5.

2. 't Hooft vortex and intersecting D-branes

The simple example we start with is two intersecting D2-branes, each lying on a different complex plane. Consider the four dimensional subspace $X^a = 0, \quad a = 5, \ldots, 9$. This subspace can be described by two complex variables $z = X^1 + iX^2$, $w = X^3 + iX^4$. Take a D2-brane lying along the z plane at $w = 0$, and another D2-brane lying along the w plane at $z=0$. The SUSY preserved by the first D2-brane is one subject to the constraint $\epsilon = \gamma^0\gamma^{12}\tilde{\epsilon}$, while the SUSY preserved by the second D2-brane is the one satisfiying $\epsilon = \gamma^0\gamma^{34}\tilde{\epsilon}$. It is easy to check that the two conditions are compatible, and only 1/4 of the whole SUSY is preserved by this configuration.

The above two D2-branes intersect at the point $(z,w)=(0,0)$. The surface can be described by the simple equation $zw=0$. It turns out that there is a family of deformations of this configuration, parametrized by one complex parameter C, $zw=C$. This was observed in [3]. For $C\neq 0$, the surface is smooth at either $z=0$ or $w=0$, but the point $(z,w)=(0,0)$ is no longer on the surface. The parameter C can be interpreted as the condensate of strings stretched between the two orthogonal D-branes.

The equation $zw=C$ has a natural interpretation in the world-volume theory of one of the D-branes. If we start with the first D-brane, then w can be regarded as a complex scalar field living on the D2-brane. The Nambu-Goto action for a static field configuration reads

$$S = -T_2\int dtd^2z\left(1+(\partial w\bar{\partial}\bar{w}-\bar{\partial}w\partial\bar{w})^2+2(\partial w\bar{\partial}\bar{w}+\bar{\partial}w\partial\bar{w})\right)^{1/2}, \qquad (2.1)$$

where T_2 is the D2-brane tension. The static energy is the minus the above integral expression with the time integral dropped out. The equation of motion derived from the above action is the one determining a minimal surface. It is easy to see that it admits holomorphic solutions, or anti-holomorphic solutions. For holomorphic solutions, the energy formula simplifies to

$$E = T_2\int d^2z(1+\partial w\bar{\partial}\bar{w}), \qquad (2.2)$$

and for consistency, it is readily checked that the a holomorphic function $w(z)$ satisfies the equation of motion derived from the simplified action.

Now substitute the equation $w=C/z$ into (2.2),

$$E = T_2\left(\int d^2z+\int d^2z\frac{|C|^2}{|z|^4}\right). \qquad (2.3)$$

The first integral gives the energy of the original D2-brane. The second integral diverges at $z=0$. Introduce a cut-off r, and demand $|z|\geq r$, the second integral results in the additional energy $T_2\pi(|C|^2/r^2)$. Now $R=|C|/r$ is naturally interpreted as the infrared cut-off for the second D2-brane lying along the complex plane w. The additional energy is precisely that of the second D2-brane. Physically, we can trust the Nambu-Born-Infeld action if the derivatives of scalar fields are not very large, this would introduce a natural short distance cut-off r on the z plane. From the above simple calculation, it appears that this cut-off can be arbitrarily small.

The interpretation of intersecting branes as a single, holomorphically embedded brane was first considered in the M theory context in [4]. The situation we are discussing here is simpler, since the target space is not only flat, but also has a trivial topology.

It is straightforward to generalize the above consideration to the case when a single D2-brane along the w plane intersects several D2-branes along the z plane. Assume these D2-branes intersect the orthogonal D2-brane at points w_i, $i = 1, \ldots, N$, the general holomorphic representation is

$$z = \sum_i \frac{C_i}{w - w_i}\,, \tag{2.4}$$

where z can be interpreted as a complex scalar field on the single D2-brane. For distinct w_i, the complex parameters C_i are arbitrary. Now interesting things happen when we push all the N D2-brane to the same point, say $w_i = 0$. It is no longer possible to keep C_i arbitrary. For instance, if $C = \sum_i C_i \neq 0$, we return to the simple case where there is only one D2-brane along the z plane. C_i must be fine tuned such that as a function of w, z has a pole of order N at $w = 0$. This is easy to see for $N = 2$, and for general N the result can be deduced inductively. As a direct check of this claim, consider the energy of the configuration $zw^N = C$. Applying formula (2.2) with the roles of z and w interchanged, we find

$$E = T_2 \left(\int d^2w + \int d^2w \frac{N^2|C|^2}{|w|^{2N+2}} \right). \tag{2.5}$$

Again introduce a cut-off $|w| \geq r$, the second integral results in $NT_2\pi(|C|^2/r^{2N})$. Now $R = |C|/r^N$ is the size cut-off for the N D2-branes, and the excessive energy is precisely that of these N D2-branes.

The Higgs field $z = C/w^N$ is well-defined on the single, orthogonal D2-brane. The physical interpretation of this solution is rather simple. If one follows a loop on the w plane around $w = 0$, one goes N loops on the z plane. That is, the coincident N D2-branes are connected from one sheet to another. It is interesting to start with these N D2-branes, and interpret w as a Higgs field. w is not well-defined as a function of z, because of the branch cut. However, we know that the world-volume theory is a nonabelian theory with gauge group $U(N)$, and a Higgs field must be a $N \times N$ matrix. Thanks to the branch cut, a diagonal matrix is easy to construct using $zw^N = C$:

$$W(z) = (\frac{C}{z})^{\frac{1}{N}} \mathrm{diag}\left(1, e^{-\frac{2\pi i}{N}}, \ldots, e^{-\frac{2\pi i(N-1)}{N}}\right). \tag{2.6}$$

Still, this Higgs field configuration is not well-defined. This is because if one follows a loop around $z = 0$, the first diagonal element will be shifted to the second, and so on. Namely $W_a(ze^{2\pi i}) = W_{a+1}(z)$.

It is well known how to resolve this problem. Perform a gauge transformation

$$\tilde{W}(z) = e^{-iA\theta} W(z) e^{iA\theta}\,, \tag{2.7}$$

where θ is the angular variable of the z plane, and A is a constant matrix satisfying

$$e^{2\pi i A} = U, \quad U_{ij} = \delta_{i,j-1}\,. \tag{2.8}$$

Thus the gauge transformation is singular, since its value after a full rotation is the shift matrix U, not 1. Now, the new Higgs field $\tilde{W}$ is single-valued. In addition to this Higgs field, there is a gauge field $A_\theta = A$. This gauge field generates a Wilson line around $z = 0$, whose effect is precisely to connect the a-th sheet to the (a+1)-th sheet.

The solution $(\tilde{W}(z), A)$ can be regarded as a vortex solution, although its energy diverges without cut-off. We propose that this vortex is a 't Hooft vortex, namely if a creation operator $\phi(z)$ is defined for the vortex, it will have the commutation relation with a Wilson loop operator as in (1.2). It is quite difficult to prove this directly. Here we only provide two intuitive arguments.

The first argument has to do with the nature of the charge carried by a vortex. Topologically, the 't Hooft vortex is possible due to the fact that $\Pi_1(SU(N)/Z_N) = Z_N$, that is, the gauge transformation induced by the presence of the vortex is defined up to a factor in Z_N when a loop around the vortex is followed. If there are N such vortices inside the loop, the group factor becomes trivial, and one can not distinguish between this configuration and a topologically trivial configuration. Thus, the charge carried by a vortex is different from a standard $U(1)$ charge. To see that this is also true of the vortex we have constructed, we need the equation describing the N D2-branes along the w plane intersecting the other N coincident D2-branes along the z plane. If the intersecting points are z_i, the holomorphic relation is

$$w^N \prod_{i=1}^{N} (z - z_i) = C \,, \tag{2.9}$$

which generalizes $zw^N = C$. When the N vortices sit at the same position, say $z_i = 0$, the above relation becomes $(zw)^N = C$. It is obvious that the different sheets on the z plane are no longer connected. We conclude that our vortex indeed carries a Z_N charge.

The second argument is indirect evidence for the exchange relation (1.2). Consider the correlation function $\langle W(C)\phi(x)\rangle$, where C is a time-like straight line in spacetime, and x is a spacetime point whose time component is $X^0 = 0$. $W(C)$ is a temporal Wilson line, the $\phi(x)$ creates a vortex at time $X^0 = 0$. If the distance between x and C is large enough, we expect the declustering behavior, namely

$$\langle W(C)\phi(x)\rangle = \langle W(C)\rangle\langle\phi(x)\rangle e^{i\alpha(C,x)} \,, \tag{2.10}$$

where the phase α might not be globally defined.

Now, adiabatically rotate C along a loop surrounding x, the phase α may jump by an amount $\Delta\alpha$. Physically, the Wilson loop is a trajectory of a quark in the fundamental representation of $SU(N)$. For a charge moving, say, on the first brane along C, it will jump to the second brane at time $X^0 = 0$, after the adiabatic rotation, since the effect of the vortex is precisely to cause the connection to the second sheet at this time. Similarly, a charge moving on the second brane will jump to the third after the rotation, and so on. Thus in general there is no reason that the correlation function

(2.10) should come back to its original value, thus the jump $\Delta\alpha$ may not be zero. However, if one executes the rotation N times, the value of the correlation function must be resumed. That is, the condition on $\Delta\alpha$ is $N\Delta\alpha = 2\pi l$ with an integer l. The minimal nonvanishing value is $\Delta\alpha = 2\pi/N$. This is precisely the phase appearing in the exchange relation (1.2). Here we must emphasize that our argument is only a plausible one. We do not have a proof that the exchange relation (1.2) is valid.

'tHooft discussed correlation function such as $\langle\phi(T)\phi^+(0)\rangle$ in [1]. The physical process involved in this correlation is a creation of a vortex at time $X^0 = 0$ which subsequently propagates to time $X^0 = T$, and is then destroyed. Our vortex has an infinite mass, thus we expect that such a correlation vanishes, since the propagator vanishes in the large mass limit. This will be checked in a large N calculation in the next section.

The discussion in this section has a straightforward generalization to D-branes with higher spatial dimensions. The case of special interest is D3-branes. Here one merely adds a common spatial dimension to the intersecting D-branes. The intersection appears as vortex line. This is a candidate for a spatial 't Hooft loop. Unlike the Wilson loop, we do not know how to write down a corresponding operator for it. Also, a spatial Wilson loop creates a state of finite mass, and it can be interpreted as trapped fundamental string within D3-branes. Our vortex line is rather similar to a temporal Wilson loop. A temporal Wilson can be regarded as traced out by a quark of infinite mass. Here, the intersection is $1+1$ dimensional, representing a spatial vortex line propagating in time. An instantaneous spatial vortex line can be constructed by a moving D3-brane which intersects N orthogonal D3-branes only at a given time.

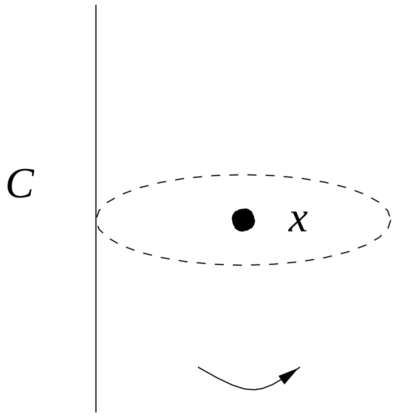

Figure 1: A Wilson loop is adiabatically brought about along a loop surrounding point x.

3. Interaction potential between a vortex and a anti-vortex at large N

To see that what we have proposed is sensible, it is useful to calculate some correlation functions or interaction potential. The latter for a vortex and an anti-vortex must be necessarily negative, if our interpretation of the configurations is meaningful. Until the recent bold conjecture of Maldacena [5], no tool for computing correlation functions in a strongly coupled gauge theory had been available. The proposed duality between a large N conformally invariant field theory and string/supergravity on an anti-de Sitter space has been precisely formulated in [6, 7], and checked to some extent. Some seeds for this conjecture appeared in earlier work [8]. There are a large number of papers written in a short period, for a partial list see the reference lists in papers cited here. We will mention only those papers which are directly related to the problem studied here.

The formalism for computing the expectation value of a Wilson loop was proposed in [9] (see also [10]). It has been applied to various large N gauge theories at zero temperature [9, 10, 11], as well as theories at a finite temperature [12, 13, 14]. The results are perfectly sensible physically. This certainly lends much support to Maldacena's conjecture.

The large N gauge theory on D2-branes is not conformally invariant. Even so, a similar duality can be valid for a certain range of energies, as analyzed in [15]. Indeed, if a finite temperature for a conformally invariant theory is switched on, the conformal invariance is lost, nevertheless the correspondence between the field theory and supergravity in the bulk should still be valid. Thus, there is no reason to exclude a nonconformally invariant theory from the conjecture.

Now, if the number of D2-branes is large enough, the gravitational background induced by these branes can no longer be ignored. Instead of analyzing the Nambu-Born-Infeld action in a flat background, as we have done in the previous section, we need to work with the action in a curved background. In the large N limit, a D2-brane intersecting with a large number of D2-branes does not have an interpretation as a smooth Higgs excitation. Indeed, the equation $zw^N = C$ does not have a well-defined large N limit. We make a change to the complex parameter such that the new equation is

$$z = \left(\frac{C}{w}\right)^N .$$

If we hold both C and z fixed and take the large N limit, we get either $z = 0$ or $z = \infty$, depending on whether $|w/C|$ is greater than or less than 1. Apparently the physically sensible choice is $z = 0$. In this case, if $C^N \sim l_s^{N+1}$, where l_s is the string scale, then the condition on $|w|$ is just $|w| > l_s$. In this case the test D2-brane intersects the stack of D2-brane precisely at $z = 0$. We shall see that this is compatible with the Nambu-Born-Infeld action in the curved background.

In the large N limit, the metric and the dilaton generated by a stack of coincident D2-branes in the near horizon region are

$$\begin{aligned} ds^2 &= \alpha' \left(\frac{U^{5/2}}{R^2} ds_3^2 + \frac{R^2}{U^{5/2}} ds_7^2\right), \\ e^\phi &= \frac{g_{YM}^2 R}{U^{5/4}}, \end{aligned} \tag{3.1}$$

where ds_3^2 is the flat Minkowski metric on D2-branes, ds_7^2 is the flat Euclidean metric on R^7, and U is the radial coordinate on R^7, $R^2 = (6\pi^2 g_{YM}^2 N)^{1/2}$. The definition of the Yang-Mills coupling is $g_{YM}^2 = g(\alpha')^{-1/2}$, where g is the string coupling constant. Since in all of our calculations, factors containing α' cancel, we set $\alpha' = 1$.

The Nambu-Born-Infeld action for a test D2-brane in the above background is

$$S = -\frac{1}{4\pi^2} \int d^3x e^{-\phi} [\det(G_{\mu\nu}\partial_\alpha X^\mu \partial_\beta X^\nu)]^{\frac{1}{2}} . \tag{3.2}$$

For a static orthogonal D2-brane, due to the rotational invariance on R^7, we can always choose its spatial coordinates to coincide with two of R^7. Other coordinates of R^7 vanish, in order for the D2-brane to intersect the source D2-branes. Denote, as in the previous section, the spatial coordinates by a complex variable w. It is easy to see that the factors depending on U inherited from the metric and dilaton in (3.1) all cancel, and the action takes the same form as in a flat space, thus the energy formula for an orthogonal D2-brane agrees with that in a flat space. This must be the case, given the BPS nature of this state. On the other hand, if the D2-brane also bends in the longitudinal direction which we denote by a complex coordinate z, the action is quite different, and is given by

$$S = -\frac{1}{4\pi^2 g_{YM}^2}\int d^3x \left(1 + 2\frac{|w|^5}{R^4}(|\partial z|^2 + |\bar{\partial} z|^2) + \frac{|w|^{10}}{R^8}(|\partial z|^2 - |\bar{\partial} z|^2)^2\right)^{\frac{1}{2}}, \qquad (3.3)$$

where we have identified U with $|w|$, and the derivatives are defined against the complex coordinates w, $\bar{w}$. It is obvious that the equations of motion derived from this action do not admit holomorphic solutions, and $z = 0$ minimizes the energy. This agrees with our observation based on solution in the flat background.

One can consider a solution of multiple parallel branes, again the solution is trivial $z = z_i$, z_i is the location of the i-th D2-brane. If, however, there is an anti-D2-brane in addition to a D2-brane, the solution is no longer constant z. To minimize the energy cost, there will develop a throat between the D2-brane and the anti-D2-brane, as in fig. 2. We will compute the energy difference between this configuration and the sum of energies of individual branes. The difference is finite but negative, confirming our expectation. In analogy to the case of the Wilson loops, we interpret the energy difference as the static potential between a vortex and an anti-vortex in the world-volume theory in the large N limit.

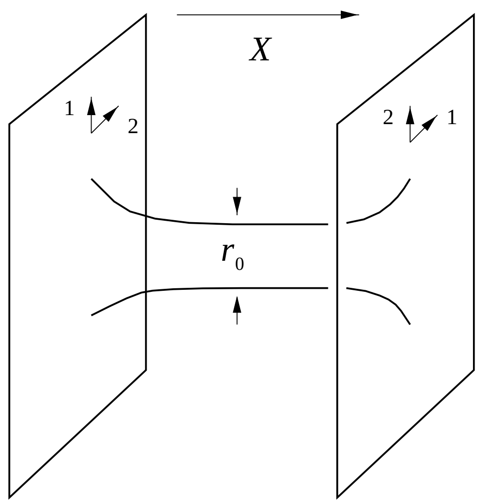

Figure 2: A pair of D2-brane and anti-D2-brane develop a throat between them in the curved background to minimize the energy cost.

Assume the location of the D2-brane is $z = L/2$, and the location of the anti-D2-brane is $z = -L/2$. From the action (3.3) which is valid locally on the w plane, we see that in order to minimize the energy, the solution will always have $\Im z = 0$. By inspecting fig. 2, it is easy to see that a good coordinate system is provided by $(x, \theta) = (\Re z, \theta)$, where θ is the angular variable on the w plane. Now, $r = |w|$ is an even function of x, due to the reflection symmetry. The appropriate boundary conditions are $r(x = \pm L/2) = \infty$.

A simple calculation leads to the energy formula

$$E = \frac{1}{4\pi^2 g_{YM}^2 R^2} \int d\theta dx r^{\frac{7}{2}} \left(1 + R^4 r^{-5} (r')^2\right)^{\frac{1}{2}} , \tag{3.4}$$

where $r' = dr/dx$. Due to translational invariance in x, the equation of motion has an integration

$$r^{\frac{7}{2}} (1 + R^4 r^{-5} (r')^2)^{-\frac{1}{2}} , = r_0^{\frac{7}{2}}, \tag{3.5}$$

where r_0 is the minimal value of r that is reached at $x = 0$, it can be interpreted as the size of the throat. r_0 is determined by the boundary condition

$$R^2 \int_{r_0}^{\infty} dr r^{-\frac{5}{2}} \left((\frac{r}{r_0})^7 - 1 \right)^{-\frac{1}{2}} = \frac{L}{2}. \tag{3.6}$$

The result is

$$r_0^{\frac{3}{2}} = \frac{4\Gamma(5/7)\Gamma(1/2) R^2}{3\Gamma(3/14) L}. \tag{3.7}$$

This is quite reasonable, since the size of throat r_0 decreases when L increases, and eventually disappears in the limit $L = \infty$.

The static energy (3.4) is divergent, and we need to subtract the bare energy of individual D-branes, which is

$$2 \times \frac{1}{4\pi^2 g_{YM}^2} \int_0^{\infty} r dr d\theta.$$

After the subtraction, the interaction potential is

$$V = E - E_0 = -\frac{r_0^2}{\pi g_{YM}^2} \left(\frac{1}{2} - \int_1^{\infty} x dx [x^{\frac{7}{2}} (x^7 - 1)^{-\frac{1}{2}} - 1] \right). \tag{3.8}$$

The number in the parenthesis is a pure number, and is positive. According to (3.7), $r_0^2 \sim (R^2/L)^{4/3} \sim (g_{YM}\sqrt{N}/L)^{4/3}$. Thus the potential scales as $N^{2/3} g_{YM}^{-2/3} L^{-4/3}$, to be contrast to the potential between a pair of heavy quark and anti-quark, which scales as $(g_{YM}^2 N)^{1/3} L^{-2/3}$ [9]. The potential between a vortex and anti-vortex falls off with L faster than that between quark and anti-quark, but grows faster with the large N.

The integral in (3.8) can be calculated. For instance it is given by an infinite series $\sum_{n\geq 1} (2n-1)!!/[(2n)!!(7n-2)]$. Since $\sum_{n\geq 1} (2n-1)!!/[(2n)!!(2n+2)] = 1/2$, the integral is smaller than $1/2$. This proves that the potential (3.8) is negative, as it should be if it represents the interaction between a vortex and an anti-vortex. This also proves our earlier claim, that a throat between the D2-brane and the anti-D2-brane develops in order to minimize the energy cost.

Another interesting exercise is to compute the correlation function $\langle\phi(T)\phi^+(0)\rangle$. We explained in the previous section that this correlation should vanish. In Maldacena's conjecture, the corresponding process in the bulk is a D2-brane moving toward the source D2-branes in a direction in R^7 that is orthogonal to its world-volume, and touching the horizon at time $X^0=0$, stuck there until time $X^0=T$, and leaving the horizon. We show that it will take an infinite amount of time for the test D2-brane to ever reach the horizon, therefore the above process is impossible.

Take, say X^5 as the transverse coordinate in which the test brane is separated from the source branes. It is a function of time. Now the induced metric components are $h_{00}=-U^{5/2}/R^2+R^2U^{-5/2}(\dot{X}^5)^2$, $h_{ij}=R^2U^{-5/2}\delta_{ij}$, where the spatial coordinates on the test brane are $X^{3,4}$. Hence $U^2=(X^3)^2+(X^4)^2+(X^5)^2$. The action reads

$$S=-\frac{1}{4\pi^2 g_{YM}^2}\int d^3x\left(1-R^4U^{-5}(\dot{X}^5)^2\right)^{\frac{1}{2}}. \tag{3.9}$$

The equation of motion derived from the above action can be solved. The simplest situation is the point $X^{3,4}=0$. The solution is

$$(X^5)^{\frac{3}{2}}(t)=a\frac{R^2}{(t-t_0)}, \tag{3.10}$$

where a is an integration constant. Thus for this point to reach $X^5=0$, it takes an infinite amount of time. Notice that this point can reach $X^5=\infty$ in finite time.

4. Vortex lines on D3-branes

As argued in sect. 2, a D3-brane intersecting a stack of D3-branes along a line is a BPS state, the line can be regarded as a vortex line in the world-volume theory of multi-D3-branes. It is a candidate for a trapped 't Hooft loop. We shall compute the static potential between a vortex line and an anti-vortex line in this section, again utilizing Maldacena's conjecture.

As in the previous section, set $\alpha'=1$. The near horizon metric is

$$ds^2=\frac{U^2}{R^2}ds_4^2+\frac{R^2}{U^2}ds_6^2, \tag{4.1}$$

where $R^2=\sqrt{4\pi gN}$, and the Yang-Mills coupling $g_{YM}^2\sim g$. The dilaton field is a constant. The metric is written as though it is a metric on spacetime $R^4\times R^6$. Due to the U dependent factors, the spacetime is actually $AdS_5\times S^5$. The supersymmetric gauge theory on N D3-branes is superconformally invariant, and the anti-de Sitter space possesses this symmetry explicitly.

The Nambu-Born-Infeld action for a test D3-brane is

$$S = -\frac{1}{(2\pi)^3 g}\int d^4x[\det(\partial_\alpha X^\mu \partial_\beta X^\nu G_{\mu\nu})]^{\frac{1}{2}} . \tag{4.2}$$

If the D3-brane is not bent in the two transverse directions $z, \bar{z}$ (which are orthogonal to the intersection line), then it is readily checked that the action is identical to that in a flat space.

For a pair of D3-brane and anti-D3-brane separated by a distance L in, say, $x = \Re z$, again we expect that a throat as in fig. 2 will develop. The integration in the action along the intersection line is trivial, and gives rise to a factor L_3 as an infrared cut-off. We use the same coordinates as in the previous section to parametrize the remaining two dimensions of the world-volume of this configuration. The static energy is given by

$$E = \frac{L_3}{(2\pi)^2 g R^2}\int dx U^3 \left(1 + \frac{R^4}{U^4}(U')^2\right)^{\frac{1}{2}} . \tag{4.3}$$

The rest of calculation goes in a similar way as in the previous section. The minimal U is

$$U_0 = \frac{2R^2}{L}\frac{\Gamma(2/3)\Gamma(1/2)}{\Gamma(1/6)} . \tag{4.4}$$

And the static potential obtained by subtracting the bare energy from (4.3) is

$$V = -\frac{L_3 U_0^2}{2\pi^2 g}\left(\frac{1}{2} - \int_1^\infty dx x[x^3(x^6-1)^{-\frac{1}{2}} - 1]\right) . \tag{4.5}$$

This potential is negative, since it can be shown that the integral in (4.5) is smaller than 1/2. Now, since $U_0^2 \sim R^4/L^2 \sim gN/L^2$, thus the potential scales as N/L^2. The dependence on g drops out, as it should for a D3-brane.

The power law L^{-2} is simply a consequence of conformal invariance. This says that the static potential per unit length along the vortex line is proportional to L^{-2}. If we assume that the potential is caused by a point-wise potential between a point on the vortex line and a point on the anti-vortex line, then this point-wise potential must scale as L^{-3}. This is a much softer potential compared with the Coulomb potential L^{-1}.

It is straightforward to generalize our calculation to the case of a finite temperature, where the relevant metric is that of the anti-de Sitter black hole, as in [12, 13, 14]. However, the integrals involved in this calculation can not be explicitly carried out. To check whether the potential is negative one has to perform some numerical calculations.

5. Vortices on M-branes

Supersymmetry preserved by a M2-brane lying along the plane (X^1, X^2) is subject to constraint $\epsilon = \gamma^{012}\epsilon$, where ϵ is a Majorana spinor with 32 components. Just as two intersecting D2-branes, two intersecting M2-branes preserve 1/4 of SUSY. For N coincident M2-branes, the equation $zw^N = c$ again is a solution to the Nambu action. Because of the branch-cut, we need to perform a singular gauge transformation. The world-volume theory is believed to be a nonabelian conformal field theory, and can be obtained from the D2-brane theory by taking the strong coupling limit.

We shall again compute the static potential between a vortex and an anti-vortex, using the anti-de Sitter background. The fundamental scale is the Planck length l_p. It drops out in our calculations so we set it to equal to 1. The near horizon metric of the N M2-branes is

$$ds^2 = \frac{r^4}{R^4} ds_3^2 + \frac{R^2}{r^2} ds_8^2 \,, \tag{5.1}$$

where r is the radial coordinate on R^8 which is transverse to the world-volume of the source branes. $R = (2^5\pi^2 N)^{1/6}$. Although the above metric is written in a form as though the topology of spacetime is $R^3 \times R^8$, due to the nontrivial r dependent factors in the metric, the spacetime is actually $AdS_4 \times S^7$.

The Nambu action for a test M2-brane is

$$S = -c_2 \int d^3x [\det(\partial_\alpha X^\mu \partial_\beta X^\nu G_{\mu\nu})]^{\frac{1}{2}} \,, \tag{5.2}$$

where c_2 is a pure number appearing in the M2-brane tension formula $T_2^M = c_2 l_p^{-3}$. Since the M2-brane always has extension in the r direction, it does not couple to the nonvanishing three-form field.

Again we use θ and $x = \Re z$ to parametrize the world-volume of the smoothly connected M2-brane and anti-M2 brane, as in fig. 2. The static energy of this configuration is

$$E = 2\pi c_2 \int dx \frac{r^4}{R^3} \left(1 + \frac{R^6}{r^6}(r')^2\right)^{\frac{1}{2}} \,. \tag{5.3}$$

Solving the equation of motion, we obtain the minimal r_0, the size of the throat,

$$r_0^2 = \frac{R^3}{L} \frac{\Gamma(3/4)\Gamma(1/2)}{\Gamma(1/4)} \,. \tag{5.4}$$

The static potential obtained by subtracting the bare energy from (5.3) is

$$V = -4\pi c_2 r_0^2 \left(\frac{1}{2} - \int_1^\infty dx x [x^4 (x^8 - 1)^{-\frac{1}{2}} - 1]\right) \,. \tag{5.5}$$

The integral is equal to $\sum_{n\geq 1} (2n-1)!!/[(2n)!!(8n-2)]$ which is smaller than $1/2$, hence the above potential is negative. The potential is proportional to r_0^2 which in turn scales as $\sqrt{N}/L$. The behavior L is compatible with conformal invariance. Just like the D3-brane case, only N figures into the formula.

Our last example is intersecting M5-branes. A M5-brane intersecting with a stack of M5-branes along three spatial dimensions preserves 1/4 of SUSY. The intersection has codimension 2 in the world-volume of coincident M5-branes. This situation is similar to, but different from the one considered in [4], since the holomorphic embedding is into a four dimensional space with a trivial topology. The near horizon geometry induced by the source M5-branes is

$$\begin{aligned} ds^2 &= \frac{U^2}{R} ds_6^2 + \frac{R^2}{r^2} ds_5^2, \\ r = U^2, & \qquad R = (\pi N)^{\frac{1}{3}} . \end{aligned} \tag{5.6}$$

r is the radial coordinate on R^5, the space transverse to M5-branes. The spacetime is $AdS_7 \times S^4$. We have set $l_p = 1$.

An orthogonal M5-brane has three spatial dimensions in common with the source M5-branes, so does the orthogonal anti-M5-brane. The integral part of the static energy in these three dimensions is trivial and gives rise to a factor V_3. The nontrivial part of the energy is

$$E = 2\pi V_3 c_5 \int dx \frac{U^5}{R^{3/2}} \left(1 + \frac{4R^3}{U^4}(U')^2\right)^{\frac{1}{2}} . \tag{5.7}$$

The size of the throat is

$$U_0 = \frac{4R^{3/2}}{L} \frac{\Gamma(3/5)\Gamma(1/2)}{\Gamma(1/10)} . \tag{5.8}$$

The bare energy of the M5-brane and the anti-M5-brane is $E_0 = 2V_3 c_5 \int 2U^3 dU d\theta$. This is to be subtracted from the energy in (5.7). We find the interaction potential

$$\begin{aligned} V &= -8\pi V_3 c_5 U_0^4 \left(\frac{1}{4} - \int_1^\infty dx x^3 [x^5 (x^{10} - 1)^{-\frac{1}{2}} - 1]\right) \\ &= -2\pi V_3 c_5 U_0^4 \frac{\Gamma(3/5)\Gamma(1/2)}{\Gamma(1/10)} . \end{aligned} \tag{5.9}$$

It is negative. The potential per unit three-volume is proportional to $U_0^4 \sim R^6/L^4 \sim N^2/L^4$. The power L^{-4} is in accord with conformal invariance. The point-wise potential between points on the intersection volumes scales as L^{-7}.

Finally, we make the following observation. In all the conformally invariant cases, the potential is independent of either the coupling or the Planck length. In the M2, D3, M5 cases, the potential is proportional to $N^{1/2}$, N, N^2 respectively. Since the potential is averaged over colors, the unaveraged result will be $N^{3/2}$, N^2 and N^3 respectively. These numbers are just the numbers of degrees of freedom on M2, D3, M5 branes.

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